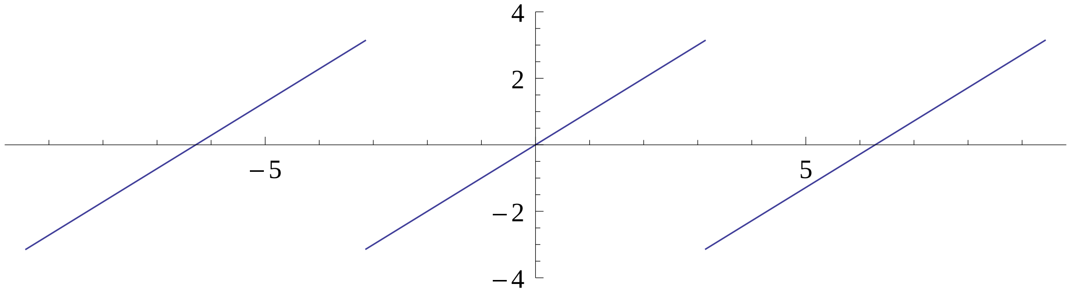
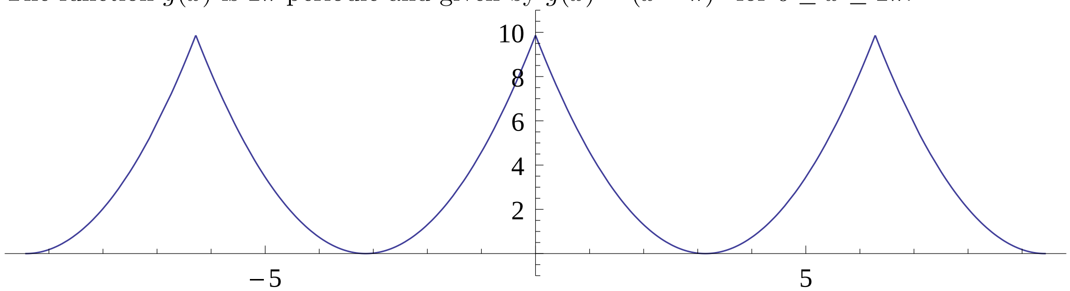
Math Homework #2

1. The function $f(x)$ graphed below has Fourier series $f(x) = \sum_{k\geq 1} \frac{2(-1)^{n+1}}{n} \sin(nx)$.



The function $g(x)$ is 2π-periodic and given by $g(x) = (x-\pi)^2$ for $0 \leq x \leq 2\pi$.



   (a) Express the function g in terms of f by scaling, horizontal shift and integration.

   (b) Use your answer from part (a) to get the Fourier coefficients of g from the Fourier series for f given above.

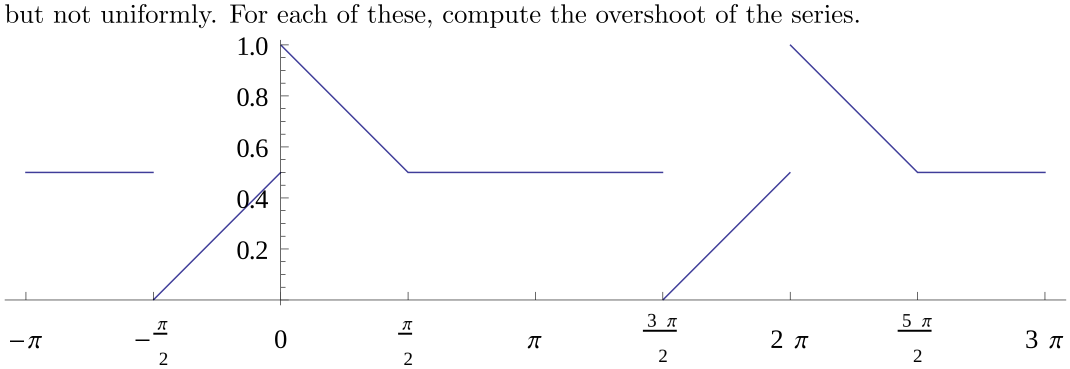
2. For each of the following functions, explain whether the Fourier series converges uniformly, pointwise, or in $L^2[0, 2\pi]$ (the series could converge w.r.t. more than one mode, or perhaps none of them). What property of the function allows you to deduce this?

   (a) $f(x) = \sin(1/x), \qquad 0 < x \leq 2\pi$

   (b) $g(x) = x^{-1/2}, \qquad 0 < x \leq 2\pi$

   (c) $h(x) = |x - \pi|, \qquad 0 < x < 2\pi$

3. For the following function, determine all x-values nearby which the Fourier series converges pointwise, but not uniformly. For each of these, compute the overshoot of the series.



4. let $g(x)$ be defined as in problem 1. What does the smoothness and decay of Fourier coefficients theorem from class tell us about the Fourier coefficient of g?

5. Define a discrete random variable X by

$$P(X = k) = \begin{cases} Ck^2 & \text{if } k \in \{1, 2, 3, 4\} \\ 0 & \text{else} \end{cases}$$

for some constant C.

(a) Calculate the value of C

(b) Calculate the mean and variance of X.

6. Suppose that the number of burglaries in a certain neighborhood is on average 4.5 per month. Let X be the number of burglaries there in some month. Explain why it is reasonable to model X with a Poisson random variable. What is the probability that $X > 10$?

7. Calculate the mean and unbiased sample variance of the following sample

$$\{1, 5, 3, 12, -3, -3, -8\}$$